

# Deep Inelastic Scattering off a Plasma with a Background Magnetic Field

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## Abstract

Using holography, we analyse deep inelastic scattering of a flavor current from a strongly coupled quark-gluon plasma with a background magnetic field. The aim is to show that how the magnetic field affects the partonic picture of the plasma. The flavored constituents of the plasma are described using D3-D7 brane model at finite temperature. We find that the presence of a background magnetic field makes it harder to detect the plasma constituents. The results will be obtained from different perspectives and we will show the consistency of the results with other approaches. In this way we will obtain a criteria for the possibility of deep inelastic scattering process in the presence of the magnetic field and the resulting changes for structure function of the plasma.

## 1 Introduction

One of the main theoretical challenges in recent years is to describe the state of matter produced in a heavy ion collision, i.e. quark-gluon plasma (QGP). Experimental results indicate that the QGP may be in a strongly coupled regime. Many of the available formalisms in quantum field theory are perturbative and appropriate for a weakly coupled system. A strongly coupled system needs non-perturbative treatment and there are not too many tools for it. One of the most important and reliable non-perturbative tools is the numerical lattice calculations which is of very

limited use in treating QGP because of the problems for incorporating the chemical potential in the formalism. An interesting approach to describe a strongly coupled system, coming from string theory, is the AdS/CFT correspondence (gauge/gravity duality) [3, 4]. AdS/CFT relates a large- $N$  gauge theory with large 't Hooft coupling  $\lambda = g^2 N$  to a weakly coupled string theory. The original correspondence was between  $N = 4$  supersymmetric Yang-Mills (SYM) theory and string theory in  $AdS_5 \times S^5$  and the finite temperature extension relates the  $N = 4$  SYM theory at finite temperature to the string theory in  $AdS_5 \times S^5$  with a black hole [5].

Deep inelastic scattering (DIS) is one of the most important experimental methods to reveal the structure of hadrons. In this process an energetic electron collides the hadron and the exchanged virtual photon probes the internal structure of the hadron. Here it is assumed that the structure of QGP can be probed by the same process. Absorption of virtual photon by the plasma is an inelastic process to reveal the partonic structure of the plasma. More specifically here absorption of the virtual space-like current excites some degrees of freedom in the plasma. Using AdS/CFT formalism the dual of the virtual photon will be constructed on the gravity side to mimic the DIS process. In the finite temperature version of the gauge/gravity formalism i.e.  $AdS$  with blackhole, the virtual photon couples to massless fields in the adjoint representation or partons associated with gluons.

To incorporate quarks i.e. fields in the fundamental representation, one way is to insert  $N_f$  D7-branes in the  $AdS_5 \times S^5$  Schwarzschild background [6]. Then the dual theory in the probe limit ( $N_f \ll N_c$ ) is the  $N = 2$  SYM theory with  $N_f$  hypermultiplets of fundamental fields. In this D3/D7 brane model the virtual photon couples to the fields in the fundamental representation or partons associated with quarks. In a common DIS experiment, photon probes the internal structure of a hadron, and in the same way the space-like gauge field living in the world volume of D7-brane probes the plasma structure. This space-like current describes an inelastic scattering of a flavor current off a strongly coupled  $N = 2$  SYM plasma. One of the advantages of using AdS/CFT correspondence in this formalism is the simplicity in describing the absorption of incoming perturbations. While it is not obvious from the boundary field theory point of view the ability of plasma to absorb all perturbations (at sufficiently high temperatures), it is almost straightforward to be deduced from the gravitational point of view and the "blackness" of blackhole. In order to describe different absorption mechanisms of a current into the plasma from a gravitational point of view, the different thermal phases of plasma in this setting needs to be concerned. It is well known that three different phases exist in the D3-D7 brane model [7] : the low temperature Minkowski, the high temperature black hole and the critical embedding. Here we will consider the first two embedding only. In the black hole embedding the D7-brane touches the black hole and have an induced horizon. It is not difficult to deduce from a gravitational point of view that the space-like flavor current living on D7-brane world volume will ultimately fall into the black hole. In the dual gauge theory the current will fluctuate into partons

and branches due to the interaction with the plasma, until it disappears completely in the plasma [1, 2].

in the Minkowski embedding the D7-brane do not touch the horizon and the brane is separated from the black hole horizon in the radial direction. As explained in [1] the space-like flavor current absorption is not obvious here, because the current on D7-brane can not fall into the black hole. A new mechanism for the flavor current absorption by plasma was introduced in [1]. This new mechanism is based on the existence of vector mesons in the D7-brane world volume of Minkowski embedding. Then the flavor current will disappear into the plasma by resonant production of the space-like vector mesons.

Here in the presence of a background magnetic field we will discuss how the kinematics of space-like current is changed at low and high energy regime of Minkowski embedding. This is done from different view points and will elaborate on the consistency them. The effect of a background magnetic field on D7-brane embedding and some of its implications was investigated in [12, 13, 14, 15].

First we obtain the dispersion relation for transverse vector mesons in the presence of a magnetic field by expanding the vector meson modes in terms of regular and normalizable modes [7, 8]. From the resulting dispersion curve, we will discuss the implications for the limiting velocity and rest mass of the vector meson in the plasma. We will also discuss how the obtained dispersion curve, affects the DIS process. Then following [1, 2], using potential analysis of the Schrodinger-like equation of vector mesons we will see that how the space-like current potential changes in the presence of magnetic field. Potential analysis for different regimes of inelastic scattering energy shows us that the absorption of the flavor current by plasma becomes harder in the presence of the magnetic field. The implications on the potential for low and high temperature phases will be analysed to show the agreement of the results with other approaches. As the resulting equations in the presence of the background magnetic field can't be solved analytically, all of the equations are solved numerically. At the first step the D7-brane embedding is solved numerically so the vector meson equations, potential analysis and other quantities are obtained in this general numerically solved approach.

Now we repeat the arguments of [1] for the conditions on the kinematics of space-like flavor current that needs to be met in order to contribute the DIS process off the plasma. Then we will check the possibility to satisfy the conditions in the presence of a background magnetic field. As explained in [1], the relevant kinematics is  $\omega \gg Q \gg T$  with  $Q^2 > 0$  and that the space-like current can excite mesons for high enough energies and small virtualities where the current and mesons are nearly light-like. The reason for smallness of the virtuality is that for large space-like one, there is a potential barrier near the Minkowski boundary. The condition that the current can penetrate in the inner region of D7-brane to excite mesons, is  $\omega \geq Q^3/T^2$ . From the dispersion curve of figure [1] in the next section, and also from potential analysis of section 3, we will see that it is harder for the current to meet the above mentioned

conditions and contribute to the DIS process. Specifically for the above relation, dispersion curve of figure [1] shows that higher energies are needed for the current to reach the light-like region which is of relevance for the DIS process discussed here. The harder DIS process in the presence of magnetic field is also consistent with the factorization picture [9, 10, 11]. The factorization picture state that the photon fluctuation into dipole ( $q\bar{q}$  pair) and the target hadron also is described by a collection of dipoles. From our results and also previous results [12, 13] it can be concluded that a magnetic field makes the  $q\bar{q}$  more bounded (so smaller in size) and from the formulation it can be seen that smaller size dipoles result in smaller cross section for the scattering events. Another condition for the possibility of DIS process is the existence of an infinite tower of equally spaced levels. It is necessary that the levels be finely spaced for fixed and large values of  $k$ . In fact as will be discussed in the next section, this can imposes a bound on the strength of the magnetic field for which the DIS can occur in a strongly coupled plasma.

The organization of the paper is as follows: in section **2** the D3/D7 model with a background magnetic field on the D7-brane is introduced and The induced metric on the D7-brane will be obtained. The the transverse vector meson equations of motion will be obtained as the second order fluctuations of the D7-brane. Using the equations of motion and expansion in normalizable modes, the dispersion curve will be plotted and the implications of the magnetic field on the curve will be discussed. Writing the equations for transverse vector mesons in a Schrodinger-like form, in **3** an analysis for the implications of the magnetic field on the potentials will be performed. The main result of this paper will be presented in section **4** by discussing the effect of magnetic field on structure functions and DIS process of plasma.

## 2 Mesons in D3/D7 model with a background magnetic field

The gravitational dual of  $N = 4$  super Yang-Mills theory at finite temperature is the decoupling limit of  $N_C$  black D3-branes, where the corresponding metric is of the form (see appendix (A)) [3, 4, 5]:

$$dS^2 = \frac{1}{2} \left( \frac{u_0 \rho}{L} \right)^2 \left( -\frac{f^2}{\tilde{f}} dt^2 + \tilde{f} dx^2 \right) + \frac{L^2}{\rho^2} (d\rho^2 + \rho^2 d\Omega_5^2) \quad (1)$$

where:

$$f(\rho) = 1 - \frac{1}{\rho^4}, \quad \tilde{f}(\rho) = 1 + \frac{1}{\rho^4} \quad (2)$$

Then by introducing the following variables:

$$\rho^2 = r^2 + R^2, \quad r = \rho \cos \theta, \quad R = \rho \sin \theta \quad (3)$$

the induced metric on D7-brane becomes:

$$dS^2 = \frac{1}{2} \left( \frac{u_0 \rho}{L} \right)^2 \left( -\frac{f^2}{\tilde{f}} dt^2 + \tilde{f} dx^2 \right) + \frac{L^2}{\rho^2} \left( (1 + \dot{R}^2) d\rho^2 + \rho^2 d\Omega_3^2 \right) \quad (4)$$

where  $R(r)$  is the profile of D7 brane embedding. Now with a constant background magnetic field ( $F_{ab}^{BG}$ ) the D7 brane action is of the form:

$$M_{ab} \equiv P[g]_{ab} + 2\pi\alpha' F_{ab} \quad (5)$$

Using the induced metric obtained in appendix [B], the second order fluctuations of field strength is:

$$\sqrt{-\det(M + 2\pi\alpha'(\delta F))} = -\sqrt{-\det(G)} \text{Tr}[(G^{-1})\delta F(G^{-1})\delta F] \quad (6)$$

Now using the induced metric  $G$  and the equations of motion for  $R$  from the Lagrangian, one obtains the D7-brane embedding in the presence of the magnetic field. In Appendix A we have plotted the brane embedding with and without a magnetic field numerically. It can be seen that the magnetic field causes the brane to be repelled away from the black hole horizon. We will see that the results obtained from other approaches, are consistent with this result for D7-brane embedding. The DBI action, quadratic in the gauge field is of the form:

$$S_{D7} = -\frac{(2\pi l_s^2)^2}{4} T_{D7} N_f \int d^8\sigma \sqrt{-G} G^{mp} G^{nq} F_{mn} F_{pq} \quad (7)$$

with metric  $G$ , the induced metric on D7-brane with a background magnetic field, defined in Eq.(B.4). The equations of motion resulting from Lagrangian (7) is:

$$\partial_m (\sqrt{-G} G^{mp} G^{nq} F_{pq}) = 0 \quad (8)$$

Here plane wave solutions will be considered:

$$A_\mu(x, r) = A_\mu(r) e^{-i\omega t + ikz} \quad (9)$$

( $\mu = t, x, y, z$ ), and the boundary condition for gauge fields:  $A_\mu(r \rightarrow \infty) = A_\mu^{(0)}$ . The profile of D7-brane embedding is obtained numerically from the equation of motion (A.5) and then substituting in the gauge field equations obtained from DBI action (8). Here only the transverse fields will be considered  $i = x, y$ , so the relevant field strength components are  $F_{ri} = \partial_r A_i$ ,  $F_{ti} = \partial_t A_i \rightarrow -i\omega A_i$ , and  $F_{zi} = \partial_z A_i \rightarrow ik A_i$ . Then the resulting equations of motion for transverse gauge fields are:

$$\ddot{A}_i + \left[ \partial_r \ln(\sqrt{-G} G^{rr} G^{ii}) \right] A_i + \frac{G^{zz}}{G^{rr}} \left( \frac{f^2}{\tilde{f}^2} \omega^2 - k^2 \right) A_i = 0 \quad (10)$$

Note that the effect of background magnetic field is contained in the D7-brane embedding which enters the above equation through the term  $\sqrt{-G}$ . The non-normalizable modes of Eq.(10) are used to calculate the structure functions of

plasma in the next section. The normalizable solutions of Eq.(10) describe the vector mesons. Here we consider the normalizable modes to see the effect of the magnetic field on their spectrum.

With the numerically solved embedding equation (Appendix A) and Fourier transforming the boundary coordinates, we obtain an eigenvalue equation for normalizable modes of vector mesons. Boundary conditions at origin and boundary results in discrete set of energy eigenvalues. If we expand transverse modes in terms of regular and normalizable modes [8, 7]:

$$A_i(\omega, k, r) = \sum_n A_n(\omega, k) \xi_n(k, r) \quad (11)$$

These normalizable modes with eigenvalues  $\omega = \omega_n(k)$  obey the following equation:

$$\begin{aligned} \partial_r^2 \xi_n(k, r) + \left[ \partial_r \ln(\sqrt{-G} G^{rr} G^{ii}) \right] \partial_r \xi_n(k, r) + \left( \frac{G^{zz}}{G^{rr}} \right) k^2 \xi_n(k, r) \\ = \left( \frac{G^{zz}}{G^{rr}} \frac{\tilde{f}}{f} \right) \omega_n(k)^2 \xi_n(k, r) \end{aligned} \quad (12)$$

Solving this eigenvalue equation for the cases with and without a background magnetic field, results in figure[1]. Some important features of the transverse vector mesons can be deduced explained below.

As can be seen from the figure, the increase in the slope results in longer path (higher energies) to reach the light-like point of the dispersion curve, where it is the region of concern here. Figure also shows that for asymptotically large  $k$ , the dispersion relation goes to a limiting velocity. Compared with the  $B = 0$  case it is evident that there is an increase in the limiting velocity due to increasing slope of the dispersion curve. This result, again, is consistent with fact that the limiting velocity is the local velocity of light at the bottom of D7-brane and that the magnetic field repels the D7-brane toward the boundary.

Another result of the presence of a magnetic field is obtained from the expansion of dispersion relation in the following approximate form [1]:

$$\omega(k) \approx M_{rest} + \frac{k^2}{2M_{kin}}, \quad M_{rest} \approx M_{(0)}(1 - 2/R_0^4), \quad M_{kin} \approx \frac{M_{(0)}}{1 - 2/R_0^4} \quad (13)$$

The dispersion curve intersection and slope of figure [1] has been increased, so from the above formula we can conclude that the rest mass of the meson in the presence of a magnetic field has been increased. This is in agreement with the results of other works [12, 13]. Their results show that mesons become more bounded in a background magnetic field.

Dispersion curve of figure 1, shows that increasing the momentum from zero drives the current to the light-like region of the meson which is the main region of concern in the DIS process. Comparing the curves with and without background

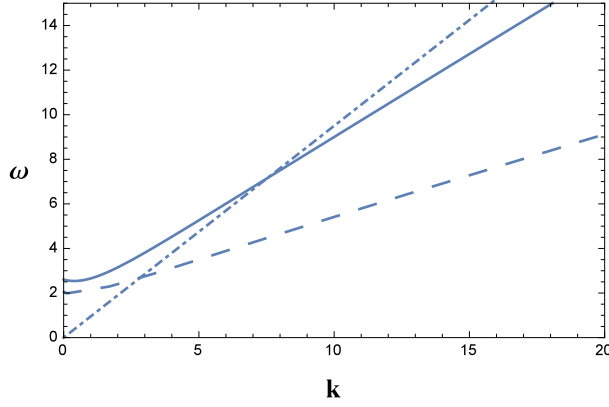


Figure 1: Dispersion curve with (solid) and without (dashed) background magnetic field. The Dot-Dashed straight line correspond to  $\omega = vk$  with  $v < 1$ .

magnetic fields shows that one needs more increase in momentum to reach the light-like region. So we see that more energy is needed to excite light-like mesons of D7-brane world volume. This point can be understood from another reasoning following [1]. The following discussion yields the criteria on the strongness of the magnetic field that allows the scattering. The kinematics of nearly light-kinematics and level spacing of meson spectrum is crucial for the DIS process to occur. The level spacing at fixed  $k$  scales as  $\delta\omega_n(k) \sim T(T/k)^{(1/3)}$ , and because the effect of background magnetic field is to increase the momentum needed for DIS process (to reach the light-like region), so increases the spacing. On the other hand the energy spacing  $\Delta\omega \sim T(M_{gap}/T)^3$  increases because of the increase in  $M_{gap}$ . Then because the necessary condition for the DIS process to occur is the condition  $\Delta\omega/\delta\omega_n \sim n^{1/3}(M_{gap}/T)^4 \gg 1$ , it can be concluded that the presence of magnetic field weakens this condition and less chance to detect partons of the plasma. So the magnetic field can be increased up to a level that the above relation holds. For higher values the conditions can't be satisfied and no partons can be detected.

### 3 Schrodinger Potentials for Vector Mesons in a Magnetic field

Here we will discuss vector mesons of D7-brane in the Minkowski embedding i.e. low temperatures, with low and high energies. It will be explained how the magnetic field affects the gauge field potential. Then it will be possible to explain the effect of magnetic field on boundary perturbations and the way these perturbations will be absorbed by the plasma. In order to write the equations of motion in a Schrodinger form and plotting the potential in a unit interval the following metric and D7-brane embedding will be used. In the following metric the boundary is located at  $u = 0$  and black hole horizon at  $u = 1$  [2]:

$$dS^2 = \frac{r_0^2}{L^2 u} (-(1-u^2)dt^2 + d\vec{x}^2) + \frac{L^2}{4u^2(1-u^2)} du^2 + L^2 d\Omega_5^2 \quad (14)$$

We have considered the effect of magnetic field on the  $\Theta(u)$  Minkowski embedding in Appendix A and the result is shown in figure [4] right. For transverse gauge fields we use the gauge  $A_u = 0$ , and the following plane wave ansatz:

$$A_\mu(x, u) = e^{-i\omega t + ikz} \bar{A}_\mu(u) \quad (15)$$

Then quadratic DBI action equations of motion for transverse gauge fields in Schrodinger form is:

$$\frac{d^2 A_i(u)}{du^2} - V(u) A_i(u) = 0 \quad (16)$$

From the dispersion curve shown in figure [1], it can be seen that low temperatures and sufficiently low momentum is the time-like region and by increasing the energy we approach the light-like region. We have plotted the potential for low energies  $\omega^2 \ll Q^2$  in figure [2] left. As can be seen the magnetic field causes a thicker potential at the boundary, so harder for the flavor current to penetrate. Also it becomes an attractive potential at a larger distance from the boundary. These again, indicate the conditions become more restricted to excite world-volume mesons.

The plot in figure [2] right, shows that at high energy and low virtuality the effect of magnetic field is to push the barrier towards the boundary and also squeezing the bottom of potential. From this we can conclude that smaller number of bound states can be formed, and so in the light-like region on concern (where virtuality changes sign), smaller number of mesons will be available to absorb the flavor current energy.

The other figure [to be added] is plotted for a low and high magnitude magnetic field. It shows that with a big magnetic field the potential behavior can do a transition. This was expected as the magnetic field causes the brane to be repelled towards the Minkowski boundary and the tip of brane will be nearer to the boundary.

The other case of interest is the low temperature and high energy [Bayona fig.7]. The effect of magnetic field is to .....[needs explanation]

## 4 Plasma Structure functions in a Magnetic field

The system of  $N_f$  D7 brane intersecting  $N_c$  D3 black brane is an  $N = 2$  gauge theory with  $N_f$  flavors and global  $U(N_f) \simeq SU(N_f) \times U(1)_q$  symmetry. The current  $J_q^\mu$  associated with conservation of quark number corresponds to diagonal subgroup  $U(1)_q$ . To mimic the deep inelastic scattering process in this holographic set up, one introduces an abelian electromagnetic gauge field  $A_\mu$  that minimally couples to  $J_q^\mu$ , i.e.  $A_\mu$  acts as the source for current  $J_q^\mu$ . So the process describing here



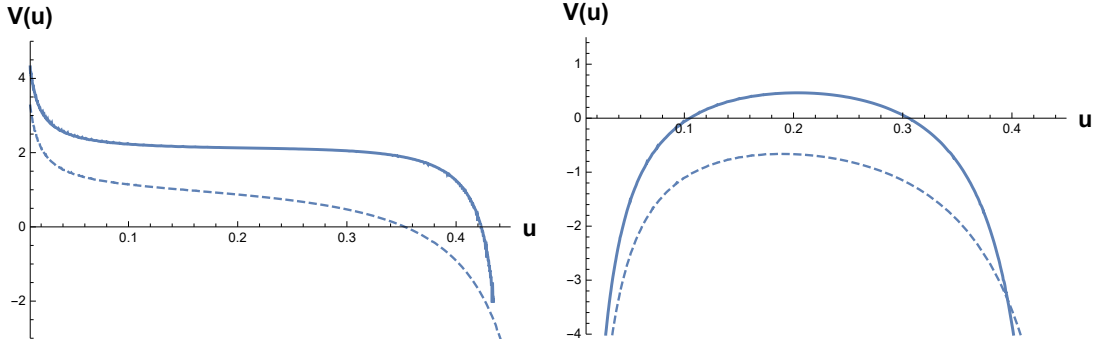


Figure 2: Low temperature potential energy for transverse flavor current, Left: for low temperature and  $\omega^2 \ll Q^2$ , Right: low energy and  $Q^2 < \omega^2$ .

is the exchange of virtual photon ( $A_\mu$ ) between the plasma and the propagating hard lepton. The deep inelastic structure functions are obtained from the following current-current correlator:

$$\Pi_{\mu\nu}(q) \equiv i \int d^4y e^{-iq \cdot y} \theta(y_0) \langle [J_q^\mu, J_q^\nu] \rangle_T \quad (17)$$

where the  $\langle \dots \rangle_T$  means thermal expectation value in the plasma. The procedure used here to obtain structure functions for the imaginary part of real-time correlators is to use:

$$\Pi_1(x, Q^2) = -\frac{N_f N_c T^2}{8} \left[ r^3 \frac{\partial_r A_i(r, \omega, k)}{A_i(r, \omega, k)} \right]_{r \rightarrow \infty} \quad (18)$$

and DIS structure functions are obtained as:

$$F_1(x, Q^2) = \frac{1}{2\pi} \text{Im} \Pi_1, \quad F_2(x, Q^2) \sim x F_1(x, Q^2) \quad (19)$$

By inserting the non-normalizable modes of gauge fields in Eq.[18] the structure function of plasma is calculated numerically for the cases with and without a background magnetic field. It can be seen from figure [??] that the magnetic field pushes the structure functions towards smaller values of Bjorken variable  $x$ . So the plasma starts to show partonic behavior in a DIS experiment, at smaller values of  $x$ . In order to interpret the changes, we rewrite the functional form of the results obtained in [1, 2] for the structure functions:

$$F_1(x, Q^2) \approx \frac{3}{4\Gamma^2(1/3)} N_f N_c T^2 \left( \frac{Q^2}{12\pi T^2 x} \right)^{(2/3)} \quad (20)$$

From the plots of figure [3], we can say that the magnetic field doesn't change the functional form of the structure function. If we relate the changes to the coefficient in the above equation, we can conclude that the background magnetic field effectively reduces the flavor degrees of freedom  $N - f$  (the flavor current doesn't couple to gluon degrees of freedom, so  $N_c$  unchanged).

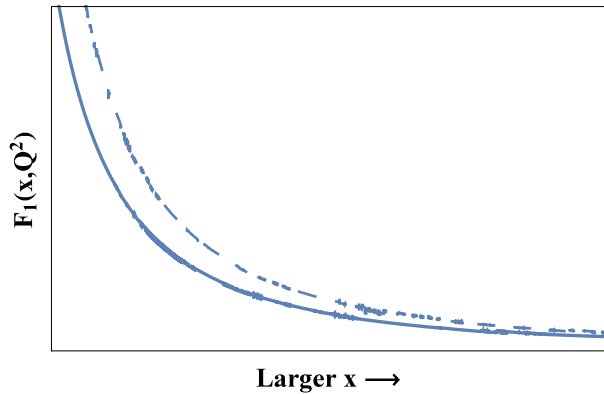


Figure 3: Structure Functions for the cases with (Solid) and without (Dashed) background magnetic field  $B$ .

## 5 SUMMARY

Here we studied the implications of a background magnetic field on a strongly coupled plasma. We showed that the presence of magnetic field causes the mesons in the plasma to become more bounded (heavier) and also the level spacing between them increased. These changes make the absorption of a flavored current harder and so less chance for DIS. The potential analysis of the Schrodinger-like equation and dispersion relations of mesons confirms the previous results. We showed that the structure function of the plasma will be pushed towards smaller  $x$  values. From these results we see that the magnetic field can change the saturation line of the plasma phase, for example by affecting the branching mechanism. Another interesting case that can be investigated from the above point of view, is the case of black hole embedding where the magnetic field has a stabilising effect.

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## Appendices

### A D7-brane embedding in different coordinates

The dual of  $N = 4SYM$  at finite temperature is the metric of the form:

$$dS^2 = \frac{u^2}{L^2} (-f(u)dt^2 + dx^2) + \frac{L^2}{u^2} \left( \frac{du^2}{f(u)} + u^2 d\Omega_5^2 \right) \quad (\text{A.1})$$

where  $L = 4\pi g_s N_c l_s^4$  is the curvature radius and  $f(u) = 1 - u_0^4/u^4$ , with  $u_0 = \pi L^2 T$  the radial position of black hole horizon.

Because we need to insert D7-branes in the above mentioned background, it is appropriate to perform the following change of radial coordinate:

$$(u_0 \rho)^2 = u^2 + \sqrt{u^4 - u_0^4} \quad (\text{A.2})$$

The above metric (A.1) transforms to the metric (14).

By the introduction of D7 brane in the above mentioned background we will have fields in the fundamental representation of  $SU(N_c)$ .

The DBI action for a D7-brane with the given  $AdS_5 \times S^5$  metric (14), and a background gauge field is of the form:

$$S_{D7} = \sqrt{\det(P[g] + 2\pi\alpha' F^{BG})} \quad (\text{A.3})$$

Then for a constant magnetic background,  $F_{xy}^{BG} = B$  the action density of the D7 brane becoms:

$$I_{D7} = \int d\rho r^3 f \sqrt{(\tilde{f}^2 + B^2)(1 + \dot{R}^2)} \quad (\text{A.4})$$

and the resulting equation of motion for  $R(r)$ :

$$\partial_r \left[ r^3 f \sqrt{\tilde{f}^2 + B^2} \frac{\partial_r R}{\sqrt{1 + (\partial_r R)^2}} \right] = r^3 \frac{\partial}{\partial R} \left( f \sqrt{\tilde{f}^2 + B^2} \right) \sqrt{1 + (\partial_r R)^2} \quad (\text{A.5})$$

This equation determines the shape of D7-brane embedding in a constant background magnetic field.

The boundaries of metric (14) corresponds to  $\rho = 1$  for black hole horizon and  $\rho \rightarrow \infty$  for Minkowski boundary. In the potential analysis of the gauge fields on the D7-brane we need a coordinate system with boundaries in a finite interval. So the following metric is introduced [Ref. Filho]:

$$ds^2 = \frac{r_0^2}{L^2 u} [-(1 - u^2)dt^2 + d\vec{x}^2] + \frac{L^2}{4u^2(1 - u^2)} du^2 + L^2 d\Omega_5^2 \quad (\text{A.6})$$

where  $r_0 = \pi L^2 T$ . The horizon is located at  $u = 1$  and the Minkowski boundary at  $u = 0$ .

Here we also need to introduce D7-branes in the space of metric (A.6). As in the previous case one needs to appropriately decompose the  $S^5$  metric:

$$d\Omega_5^2 = d\Theta^2 + \sin^2 \Theta d\Omega_3^2 + \cos^2 \Theta d\varphi^2 \quad (\text{A.7})$$

Choosing  $\varphi = 0$  and  $\Theta = \Theta(u)$  then the induced metric for D7-brane becomes:

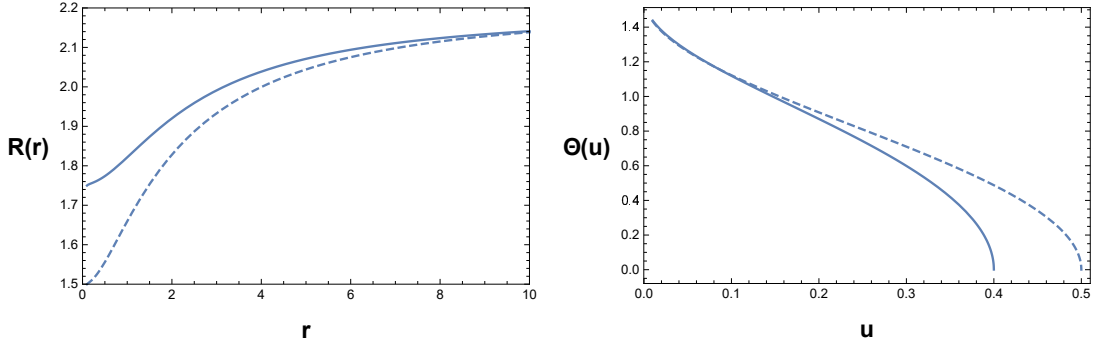


Figure 4: D7 brane embedding with (Solid) and without (Dashed) background magnetic field, Left: embedding for  $R(r)$  obtained from Eq.(4), Right: embedding for  $\Theta(u)$  obtained from Eq.(A.8) .

$$dS^2 = \frac{r_0^2}{L^2 u} (-(1-u^2)dt^2 + d\vec{x}^2) + L^2 \left( \frac{1}{4u^2(1-u^2)} + \Theta'^2 \right) du^2 + L^2 \sin^2 \Theta d\Omega_3^2 \quad (\text{A.8})$$

Again by writing the DBI action in the form of Eq.(A.3) and the equations of motion for  $\Theta(u)$ , the D7-brane embedding in this metric in the presence of a background magnetic field will be obtained. Plots of the embedding with and without a magnetic field is shown in figure [4].

## B Induced metric of D7-brane

In this appendix, following [?], the induced metric on the D7-brane in the presence of a background magnetic field is investigated.

The induced metric on D7-brane includes original induced metric  $P[g]_{ab}$  and the background  $F_{ab}$ :

$$M_{ab} \equiv P[g]_{ab} + 2\pi\alpha' F_{ab} \quad (\text{B.1})$$

Denoting the fluctuation field strength by  $\delta F_{ab}$  and keeping  $\delta F$  to second order, and decomposing the inverse metric into symmetric and anti-symmetric part,  $M^{-1} \equiv (M^{-1})^s + (M^{-1})^a$ , then we have:

$$\sqrt{-\det(M + 2\pi\alpha'(\delta F))} = -\sqrt{-\det(M)} \times \frac{(2\pi\alpha')^2}{4} \text{Tr}[(M^{-1})^s \delta F (M^{-1})^s \delta F] \quad (\text{B.2})$$

the front factor decomposes to:

$$\sqrt{-\det(M)} = \sqrt{-\det(((M^{-1})^s)^{-1})} \times \gamma, \quad \gamma = \det[1 + ((M^{-1})^s)^{-1} (M^{-1})^a] \quad (\text{B.3})$$

Now by introducing metric  $G$ :

$$G_{ab} = \mu_7^{1/2} \gamma^{1/2} (2\pi\alpha') [((M^{-1})^s)^{-1}]_{ab} \quad (\text{B.4})$$

The fluctuation Lagrangian takes the following form:

$$\sqrt{-\det(M + 2\pi\alpha'(\delta F))} = -\sqrt{-\det(G)} \text{Tr}[(G^{-1})\delta F(G^{-1})\delta F] \quad (\text{B.5})$$

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